HYPER VERSION

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Propagators for massive symmetric tensor and p-forms in AdS_{d+1}

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ABSTRACT: We construct propagators in euclidean AdS_{d+1} space-time for massive p-forms and massive symmetric tensors.

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1. Introduction

Calculations of correlation functions in type-IIB supergravity on $AdS_5 \times S^5$ have been performed extensively. These calculations study the strong coupling dynamics of $\mathcal{N} = 4$ SU(N) Yang-Mills theory for large N as a result of the AdS/CFT correspondence conjectured in [1, 2, 3]. For calculations of 4-point functions, bulk to bulk Green's functions, describing propagation between two points in the interior of AdS, are required. Propagators for scalar fields have been discussed in [4], and propagators for massless and massive gauge bosons were obtained in [5]. In [6], a new method for calculation of these propagators was discussed in which ansatze for the bi-tensor propagators were used which naturally separated the gauge invariant parts from the gauge artifacts. The gauge artifacts did not contribute if sources of the fields were conserved currents. Thus gauge fixing was unnecessary.

In this paper, we use the same method as in [6] but now for massive p-forms and the massive symmetric tensor fields. Since the fields are massive, there is no gauge invariance which guarantees that the sources are conserved. However, as we will see, using a similar ansatze for the p-forms (writing the propagator as a physical part and a pure gauge) in the massive case considerably simplifies the calculations. The pure gauge part is annihilated by the Maxwell operator and just appears multiplying m^2 in the equation of motion. For the massive symmetric tensor, the pure gauge part of the ansatz corresponds to diffeomorphisms at z (where the propagator describes propagation from z to w). However, for our propagator to have symmetry under the exchange $z \leftrightarrow w$, we need to add a term which corresponds to diffeomorphisms with at w. This term will not be annihilated by the "wave operator". However, writing the propagator in this form still simplifies the calculation considerably.

The paper is organized as follows. In section 2 we discuss the massless 2-form case as a warm-up exercise (this case was discussed in [8]). We also find the propagators for the massive anti-symmetric tensor. We generalize the calculation of section 2 to p-forms in section 3. In section 4, we perform the calculation for the propagator of the massive symmetric tensor. In section 5, we check the short-distance limit of our results and find that they match with the short distance limit of the corresponding propagator in flat space. We end with a summary of our results in section 6.

We will work in euclidean AdS_{d+1}, which can be regarded as an upper half space $z_0 > 0$ in a space with coordinates $z_0, z_1, \ldots, z_d$, and metric,

$$ds^2 = \frac{1}{z_0^2}\left(dz_0^2 + \sum_{i=1}^{d} dz_i^2\right). \tag{1.1}$$

The AdS_{d+1} scale has been set to unity and the metric describes a space with a constant negative curvature $R = -d(d+1)$. We will introduce a chordal distance u in terms of which invariant functions and tensors on AdS_{d+1} can be expressed most simply:

$$u \equiv \frac{(z-w)^2}{2z_0 w_0}, \tag{1.2}$$

where $(z-w)^2 = \delta_{\mu\nu}(z-w)_\mu(z-w)_\nu$ is the "flat euclidean distance". We will construct basic bi-tensors by taking derivatives with respect z or w of the bi-scalar variable u. These are given by[1]

$$\partial_\mu \partial_{\nu'} u = -\frac{1}{z_0 w_0}\left[\delta_{\mu\nu'} + \frac{1}{w_0}(z-w)_\mu \delta_{\nu' 0} + \frac{1}{z_0}(w-z)_{\nu'}\delta_{\mu 0} - u\delta_{\mu 0}\delta_{\nu' 0}\right], \tag{1.3}$$

and $\partial_\mu u \partial_{\nu'} u$ with

$$\begin{aligned} \partial_\mu u &= \frac{1}{z_0}\left[\frac{(z-w)_\mu}{w_0} - u\delta_{\mu 0}\right], \\ \partial_{\nu'} u &= \frac{1}{w_0}\left[\frac{(w-z)_{\nu'}}{z_0} - u\delta_{\nu' 0}\right]. \end{aligned} \tag{1.4}$$

[1] $\partial_\mu = \partial/\partial z^\mu$, $\partial_{\mu'} = \partial/\partial w^{\mu'}$.

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We will need certain properties of derivatives of u, most of which were derived in [7] and which we list here.

$$\begin{aligned}
\Box u &= D^\mu \partial_\mu u = (d+1)(1+u)\,, \\
D^\mu u\, \partial_\mu u &= u(2+u)\,, \\
D_\mu \partial_\nu u &= g_{\mu\nu}(1+u)\,, \\
(D^\mu u)\,(D_\mu \partial_\nu \partial_{\nu'} u) &= \partial_\nu u \partial_{\nu'} u\,, \\
(D^\mu u)\,(\partial_\mu \partial_{\nu'} u) &= (1+u)\partial_{\nu'} u\,, \\
(D^\mu \partial_{\mu'} u)\,(\partial_\mu \partial_{\nu'} u) &= g_{\mu'\nu'} + \partial_{\mu'} u \partial_{\nu'} u\,, \\
D_\mu \partial_\nu \partial_{\nu'} u &= g_{\mu\nu} \partial_{\nu'} u\,.
\end{aligned} \tag{1.5}$$

2. Antisymmetric tensor

The equation of motion for an anti-symmetric tensor field in AdS_{d+1} is

$$\frac{1}{2} D^\mu \partial_{[\mu} A_{\nu\alpha]} - m^2 A_{\nu\alpha} = J_{\nu\alpha}\,. \tag{2.1}$$

The covariant derivative is with respect to the AdS metric. The Maxwell operator is normalized such that $\Box A_{\mu\nu}$ appears with coefficient 1 in the equation of motion. We look for solutions of the form

$$A_{\mu\nu} = \int d^{d+1}w \sqrt{g}\, G_{\mu\nu}{}^{\mu'\nu'}(z,w) J_{\mu'\nu'}(w)\,, \tag{2.2}$$

with bi-tensor propagator $G_{\mu\nu}{}^{\mu'\nu'}(z,w)$. Using this expression for $A_{\mu\nu}$ in eq. (2.1), we obtain an equation for the propagator $G_{\mu\nu}{}^{\mu'\nu'}(z,w)$:[2]

$$\frac{1}{2} D^\mu \partial_{[u} G_{\nu\alpha]}{}^{\nu'\alpha'}(z,w) - m^2 G_{\nu\alpha}{}^{\nu'\alpha'}(z,w) = -\delta(z,w) \left(g_\nu{}^{\nu'} g_\alpha{}^{\alpha'} - g_\alpha{}^{\nu'} g_\nu{}^{\alpha'} \right). \tag{2.3}$$

The case $m = 0$. We will first look at the case $m = 0$. In this case, there is a gauge invariance of the form $A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$ which implies that the current $J^{\mu\nu}$ is conserved. The equation for $G_{\mu\nu}{}^{\mu'\nu'}(z,w)$ is:

$$\frac{1}{2} D^\mu \partial_{[\mu} G_{\nu\alpha]}{}^{\nu'\alpha'}(z,w) = -\delta(z,w) \left(g_\nu{}^{\nu'} g_\alpha{}^{\alpha'} - g_\alpha{}^{\nu'} g_\nu{}^{\alpha'} \right) + \partial^{\nu'} \Lambda_{\nu\alpha}{}^{\alpha'} - \partial^{\alpha'} \Lambda_{\nu\alpha}{}^{\nu'}, \tag{2.4}$$

where the second term on the right hand side gives zero when integrated with conserved currents. We will introduce two bi-tensors,

$$\begin{aligned}
T_{\mu\nu}{}^{\mu'\nu'} &= \left(\partial_\mu \partial^{\mu'} u \partial_\nu \partial^{\nu'} u - \partial_\mu \partial^{\nu'} u \partial_\nu \partial^{\mu'} u \right), \qquad (2.5) \\
S_{\mu\nu}{}^{\mu'\nu'} &= \left(\partial_\mu \partial^{\mu'} u \partial_\nu u \partial^{\nu'} u - \partial_\mu \partial^{\nu'} u \partial_\nu u \partial^{\mu'} u - \partial_\nu \partial^{\mu'} u \partial_\mu u \partial^{\nu'} u + \partial_\nu \partial^{\nu'} u \partial_\mu u \partial^{\mu'} u \right).
\end{aligned}$$

[2] $[\ldots]$ denotes anti-symmetrization with strength 1

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These are the only two bi-tensors which are anti-symmetric under $\mu \leftrightarrow \nu$ and $\mu' \leftrightarrow \nu'$. We will choose the following ansatz for $G_{\mu\nu}{}^{\mu'\nu'}(z,w)$:

$$G_{\nu\alpha}{}^{\nu'\alpha'}(z,w) = T_{\nu\alpha}{}^{\nu'\alpha'}F(u) + \partial_\nu L_\alpha{}^{\nu'\alpha'} - \partial_\alpha L_\nu{}^{\nu'\alpha'} .$$

The second term is a pure gauge and is annihilated by the Maxwell operator:

$$\begin{aligned}\partial_{[\mu}G_{\nu\alpha]}{}^{\nu'\alpha'} &= T_{[\nu\alpha}{}^{\nu'\alpha'}\partial_{\mu]}F'(u)\\ &= \left(\partial_{[\nu}\partial^{\nu'}u\partial_\alpha\partial^{\alpha'}u\partial_{\mu]}u - \partial_{[\nu}\partial^{\alpha'}u\partial_\alpha\partial^{\nu'}u\partial_{\mu]}u\right)F'(u) .\end{aligned}$$

In eq. (2.4), we need an expression for $D^\mu\partial_{[\mu}G_{\nu\alpha]}{}^{\nu'\alpha'}$:

$$\begin{aligned}D^\mu\partial_{[\mu}G_{\nu\alpha]}{}^{\nu'\alpha'} &= D^\mu(T_{[\nu\alpha}{}^{\nu'\alpha'}\partial_{\mu]}F'(u))\\ &= (\partial^{\nu'}u)\partial_\alpha\partial^{\alpha'}u\partial_{\mu]}uF'(u) + \partial_{[\nu}\partial^{\nu'}u(D^\mu\partial_\alpha\partial^{\alpha'}u)\partial_{\mu]}uF'(u) +\\ &\quad + \partial_{[\nu}\partial^{\nu'}u\partial_\alpha\partial^{\alpha'}u(D^\mu\partial_{\mu]}u)F'(u) + \partial_{[\nu}\partial^{\nu'}u\partial_\alpha\partial^{\alpha'}u\partial_{\mu]}uD^\mu uF''(u) -\\ &\quad - \nu' \leftrightarrow \alpha' .\end{aligned} \tag{2.6}$$

Various terms in eq. (2.6) are simplified using properties of derivatives of u (see eqs. (1.5)):

$$\begin{aligned}(D^\mu\partial_{[\nu}\partial^{\nu'}u)\partial_\alpha\partial^{\alpha'}u\partial_{\mu]}uF'(u) &= \left(\partial^{\nu'}u\partial_\nu\partial^{\alpha'}u\partial_\alpha u - \partial^{\nu'}u\partial_\alpha\partial^{\alpha'}u\partial_\nu u\right)(d-1)F'(u) ,\\ \partial_{[\nu}\partial^{\nu'}u(D^\mu\partial_\alpha\partial^{\alpha'}u)\partial_{\mu]}uF'(u) &= -\left(\partial^{\alpha'}u\partial_\nu\partial^{\nu'}u\partial_\alpha u - \partial^{\alpha'}u\partial_\nu u\partial_\alpha\partial^{\nu'}u\right)(d-1)F'(u) ,\\ \partial_{[\nu}\partial^{\nu'}u\partial_\alpha\partial^{\alpha'}u(D^\mu\partial_{\mu]}u)F'(u) &= T_{\nu\alpha}{}^{\nu'\alpha'}(d-1)(1+u)F'(u) ,\\ \partial_{[\nu}\partial^{\nu'}u\partial_\alpha\partial^{\alpha'}u\partial_{\mu]}uD^\mu uF''(u) &= u(2+u)T_{\nu\alpha}{}^{\nu'\alpha'}F'' - (1+u)S_{\alpha\nu}{}^{\alpha'\nu'}F''(u) .\end{aligned}$$

Collecting all the terms together,

$$\begin{aligned}D^\mu\partial_{[\mu}G_{\nu\alpha]}{}^{\nu'\alpha'} &= 2\Big(u(2+u)F'' + (d-1)(1+u)F'\Big)T_{\nu\alpha}{}^{\nu'\alpha'} -\\ &\quad - 2\Big((d-1)F' + (1+u)F''\Big)S_{\alpha\nu}{}^{\alpha'\nu'} .\end{aligned} \tag{2.7}$$

Using AdS invariance and the fact that $\Lambda_{\nu\alpha;\alpha'}$ is anti-symmetric in ν and α, $\Lambda_{\nu\alpha}{}^{\alpha'}$ has to be of the form:

$$\Lambda_{\nu\alpha}{}^{\alpha'} = \left(\partial^{\alpha'}\partial_\nu u\partial_\alpha u - \partial^{\alpha'}\partial_\alpha u\partial_\nu u\right)\Lambda(u) , \tag{2.8}$$

where Λ is a scalar function of u. Then,

$$\partial^{\nu'}\Lambda_{\nu\alpha}{}^{\alpha'} - \partial^{\alpha'}\Lambda_{\nu\alpha}{}^{\nu'} = -2\Lambda T_{\nu\alpha}{}^{\nu'\alpha'} + \Lambda' S_{\alpha\nu}{}^{\alpha'\nu'} .$$

Using these expressions, eq. (2.4) becomes (for z and w separate)

$$\begin{aligned}&\Big(u(2+u)F'' + (d-1)(1+u)F' + 2\Lambda\Big)T_{\nu\alpha}{}^{\nu'\alpha'} -\\ &\quad - \Big((d-1)F' + (1+u)F'' - \Lambda'\Big)S_{\alpha\nu}{}^{\alpha'\nu'} = 0 .\end{aligned}$$

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Setting the scalar coefficients of the two independent tensors to zero, we obtain

$$u(2+u)F'' + (d-1)(1+u)F' = -2\Lambda\,, \tag{2.9}$$

$$(1+u)F'' + (d-1)F' = \Lambda'\,. \tag{2.10}$$

eq. (2.10) can be integrated to give[3]

$$\Lambda = (d-2)F + (1+u)F'\,, \tag{2.11}$$

which can be substituted into (2.9):

$$u(2+u)F'' + (d+1)(1+u)F' + 2(d-2)F = 0\,. \tag{2.12}$$

This is just the invariant equation

$$(\Box - \mu^2)F(u) = 0\,, \tag{2.13}$$

for the propagator of a scalar field of $\mu^2 = -2(d-2)$. A scalar field of mass m^2 is characterized by two possible scale dimensions,

$$\Delta_\pm = \frac{d}{2} \pm \frac{1}{2}\sqrt{d^2+4\mu^2}\,. \tag{2.14}$$

For our propagator to have the fastest fall off as $u \to \infty$, in the following solution, we will choose $\Delta = \Delta_+$.

$$\begin{aligned} F(u) &= \tilde{C}_\Delta (2u^{-1})^\Delta F\left(\Delta, \Delta - \frac{d}{2} + \frac{1}{2}; 2\Delta - d + 1; -2u^{-1}\right)\,, \\ \tilde{C}_\Delta &= \frac{\Gamma(\Delta)\Gamma(\Delta - d/2 + 1/2)}{(4\pi)^{(d+1)/2}\Gamma(2\Delta - d + 1)}\,, \\ \Delta &= \frac{d}{2} + \sqrt{\frac{d^2}{4} - 2(d-2)}\,, \end{aligned} \tag{2.15}$$

where F is the standard hypergeometric function ${}_2F_1$. The constant C_Δ is chosen such that as $u \to 0$, $F(u)$ matches on to the flat space case (this is discussed in more detail in section 6).

The case $m \neq 0$. We will now consider the massive case. In this case, there is no gauge invariance and the current $J^{\mu\nu}$ is not necessarily conserved. We will still use the same ansatz,

$$G_{\nu\alpha}{}^{\nu'\alpha'} = F(u)T_{\nu\alpha}{}^{\nu'\alpha'} + \left(\partial_\nu L_\alpha{}^{\nu'\alpha'} - \partial_\alpha L_\nu{}^{\nu'\alpha'}\right)\,. \tag{2.16}$$

[3]We consistently ignore constants of integration since we want our propagators to go to zero as $u \to \infty$.

Using AdS invariance, we can write $L_{\alpha;\nu'\alpha'}$ as

$$L_\alpha{}^{\nu'\alpha'} = \left(\partial_\alpha \partial^{\alpha'} u \partial^{\nu'} u - \partial_\alpha \partial^{\nu'} u \partial^{\alpha'} u\right) L(u)\,, \tag{2.17}$$

where $L(u)$ is a scalar function. $D^\mu \partial_{[\mu} G_{\nu\alpha];\nu'\alpha'}$ is still given by eq. (2.7) since the Maxwell operator annihilates the second term in eq. (2.16). This term is

$$\partial_\nu L_\alpha{}^{\nu'\alpha'} - \partial_\alpha L_\nu{}^{\nu'\alpha'} = 2LT_{\alpha\nu}{}^{\alpha'\nu'} + L' S_{\alpha\nu}{}^{\alpha'\nu'}\,. \tag{2.18}$$

Eq. (2.3) then becomes (for z and w separate)

$$\begin{aligned}(u(2+u)F'' + (d-1)(1+u)F')\, T_{\nu\alpha;\nu'\alpha'} - ((d-1)F' + (1+u)F'')\, S_{\alpha\nu;\alpha'\nu'} - \\ - m^2 F T_{\nu\alpha;\nu'\alpha'} - m^2(2LT_{\alpha\nu;\alpha'\nu'} + L' S_{\alpha\nu;\alpha'\nu'}) = 0\,.\end{aligned}$$

Setting the coefficient of independent tensors to zero, we get a system of two coupled differential equations:

$$u(2+u)F'' + (d-1)(1+u)F' - m^2 F - 2m^2 L = 0\,, \tag{2.19}$$

$$-(d-1)F' - (1+u)F'' - m^2 L' = 0\,. \tag{2.20}$$

Eq. (2.20) can be readily integrated to give

$$L = -\frac{1}{m^2}((d-2)F + (1+u)F')\,. \tag{2.21}$$

Substituting in eq. (2.19), we find an uncoupled differential equation for $F(u)$

$$u(2+u)F'' + (d+1)(1+u)F' + (-m^2 + 2(d-2))F = 0\,. \tag{2.22}$$

This is again the invariant equation

$$(\Box - \mu^2)F(u) = 0\,, \tag{2.23}$$

for the propagator of a scalar field of $\mu^2 = (m^2 - 2(d-2))$.

3. p-forms

The preceding calculation of the propagator of an anti-symmetric tensor can be generalized to p-form propagators. The equations of motion for a p-form field is

$$\frac{1}{p!} D^\alpha \partial_{[\alpha} A_{\mu_1\mu_2\cdots\mu_p]} - m^2 A_{\mu_1\mu_2\cdots\mu_p} = 0\,. \tag{3.1}$$

We will assume a solution of the form

$$A_{\mu_1\mu_2\cdots\mu_p} = \int d^{d+1}w \sqrt{g}\, G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}(z,w) J_{\mu_1'\mu_2'\cdots\mu_p'}(w)\,,$$

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with bi-tensor propagator $G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}(z,w)$. The propagator satisfies the equation:

$$\frac{1}{p!}D^\alpha\partial_{[\alpha}G_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'}(z,w) - m^2 G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}(z,w) = \\ = -\delta(z,w)g_{[\mu_1}{}^{\mu_1'}g_{\mu_2}{}^{\mu_2'}\cdots g_{\mu_p]}{}^{\mu_p'}\,. \quad (3.2)$$

There are two independent tensors which have the right anti-symmetry property under exchange of various indices. These are

$$\begin{aligned} T_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} &= \partial_{[\mu_1}\partial^{\mu_1'}u\partial_{\mu_2}\partial^{\mu_2'}u\cdots\partial_{\mu_p]}\partial^{\mu_2'}u\,,\\ S_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} &= \partial_{[\mu_1}u\partial^{[\mu_1'}u\partial_{\mu_2}\partial^{\mu_2'}u\cdots\partial_{\mu_p]}\partial^{\mu_2']}u\,. \end{aligned} \quad (3.3)$$

Generalizing the ansatz of the last section for $G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}(z,w)$, we try a solution of the form

$$G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} = F(u)T_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} + \partial_{[\mu_1}L_{\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'}\,, \quad (3.4)$$

where

$$L_{\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} = \Big(\partial^{[\mu_1'}u\partial_{\mu_2}\partial^{\mu_2'}u\cdots\partial_{\mu_p}\partial^{\mu_p']}u\Big)L(u)\,. \quad (3.5)$$

Then,

$$\partial_{[\mu_1}L_{\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} = pLT_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} + L'S_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}\,, \quad (3.6)$$

and

$$\partial_{[\alpha}G_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} = F'\partial_{[\alpha}uT_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'}\,. \quad (3.7)$$

Notice that the second term in $G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}$ (of the form ∂L does not contribute). Acting with D^α, we get

$$\begin{aligned} D^\alpha\partial_{[\alpha}G_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} = {} & F''D^\alpha u\partial_{[\alpha}uT_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} + \\ & + F'(D^\alpha\partial_{[\alpha}u)T_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} + \\ & + F'\partial_{[\alpha}uD^\alpha T_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'}\,. \end{aligned}$$

Various terms appearing on the r.h.s. of the above equation can be simplified by using eq. (1.5):

$$\begin{aligned} D^\alpha u\partial_{[\alpha}uT_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} &= p!\,\Big(u(2+u)T_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} - (1+u)S_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}\Big),\\ (D^\alpha\partial_{[\alpha}u)T_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} &= p!((d+1-p)(1+u)T_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'})\,,\\ \partial_{[\alpha}uD^\alpha T_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} &= -p!(d+1-p)S_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}\,. \end{aligned}$$

So (3.2) becomes (for z and w separate),

$$\begin{aligned} &\frac{1}{p!}D^\alpha\partial_{[\alpha}G_{\mu_1\mu_2\cdots\mu_p]}{}^{\mu_1'\mu_2'\cdots\mu_p'} - m^2G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} = \\ &\Big(u(2+u)F'' + (d+1-p)F' - m^2F - pm^2L\Big)T_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} - \\ &- \Big(((1+u)F'' + (d+1-p)F') - m^2L'\Big)S_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} = 0\,, \end{aligned}$$

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which implies

$$u(2+u)F'' + (d+1-p)F' - m^2F - pm^2L = 0\,, \tag{3.8}$$

$$(1+u)F'' + (d+1-p)F' + m^2L' = 0\,. \tag{3.9}$$

Eq. (3.9) can be integrated to give

$$L = -\frac{1}{m^2}((d-p)F + (1+u)F')\,, \tag{3.10}$$

which can then be substituted in eq. (3.8):

$$u(2+u)F'' + (d+1)F' + (-m^2 + p(d-p))F = 0\,. \tag{3.11}$$

This is the equation for the propagator of the scalar field of $\mu^2 = m^2 - p(d-p)$. The solution to this equation is given by eq. (2.15) with

$$\Delta = \frac{d}{2} + \frac{1}{2}\sqrt{d^2 + 4m^2 - 4p(d-p)}\,.$$

This result agrees with [9] for the case $m = 0$.

4. Massive symmetric tensor

We will begin by reviewing the equations of motion for the massless graviton. The propagator for the graviton was obtained in [6]. The gravitational action is

$$I_g = \frac{1}{2\kappa^2}\int d^{d+1}z\sqrt{g}(R-\Lambda) + S_m\,, \tag{4.1}$$

where S_m is the matter action. The equations of motion are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R-\Lambda) = T_{\mu\nu}\,. \tag{4.2}$$

$T_{\mu\nu}$ is the stress-energy tensor. For $\Lambda = -d(d-1)$ and $T_{\mu\nu} = 0$, we obtain euclidean AdS space with $R = -d(d+1)$. In the presence of a matter source, the metric will no longer be the AdS metric $g_{\mu\nu}$. We will denote the fluctuations about $g_{\mu\nu}$ by $h_{\mu\nu}$. An equivalent form of the equation is

$$R_{\mu\nu} + dg_{\mu\nu} = \tilde{T}_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{d-1}g_{\mu\nu}T_\sigma{}^\sigma\,. \tag{4.3}$$

The linearized equations of motion for $h_{\mu\nu}$ are

$$-D^\sigma D_\sigma h_{\mu\nu} - D_\mu D_\nu h_\sigma{}^\sigma + D_\mu D^\sigma h_{\sigma\nu} + D_\nu D^\sigma h_{\mu\sigma} - 2(h_{\mu\nu} - g_{\mu\nu}h_\sigma{}^\sigma) = 2\tilde{T}_{\mu\nu}\,. \tag{4.4}$$

All covariant derivatives and contractions are with respect to $g_{\mu\nu}$.

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For the case of massive symmetric tensor, $S_{\mu\nu}$, a consistent action with coupling to gravity is not known. For example, consider a Kaluza-Klein reduction of a theory of gravity in D dimensional space-time to $D-1$ dimensional space time. We get an infinite tower of massive symmetric tensor fields in $D-1$ dimensions. It was shown in [10] that it is impossible to consistently truncate this theory to a finite number of symmetric tensor fields. However, the quadratic part of the action (which is what we need for calculation of the propagator) was given in [11, 12].

$$I = \int d^{d+1}z\sqrt{g}\Big(\frac{1}{4}D_\mu S D^\mu S - \frac{1}{4}D_\mu S_{\nu\alpha}D^\mu S^{\nu\alpha} - \frac{1}{2}D^\mu S_{\mu\nu}D^\nu S + \frac{1}{2}D_\mu S_{\nu\alpha}D^\alpha S^{\nu\mu} + \\ + \frac{1}{2}S_{\mu\nu}S^{\mu\nu} - \frac{d-1}{2}S^2 - \frac{m^2}{4}S_{\mu\nu}S^{\mu\nu} + \frac{m^2}{4}S^2 - S_{\mu\nu}J^{\mu\nu}\Big),$$

where $S = g_{\mu\nu}S^{\mu\nu}$. From this action, we can derive the following equation of motion:

$$-\frac{1}{2}D^\sigma D_\sigma S_{\mu\nu} - \frac{1}{2}D_\mu D_\nu S + \frac{1}{2}D_\mu D^\sigma S_{\sigma\nu} + \frac{1}{2}D_\nu D^\sigma S_{\sigma\mu} - S_{\mu\nu} + \frac{1}{2}g_{\mu\nu}D^\sigma D_\sigma S - \\ -\frac{1}{2}g_{\mu\nu}D_\mu D_\sigma S^{\sigma\mu} - \frac{d-1}{2}g_{\mu\nu}S - \frac{m^2}{2}S_{\mu\nu} + \frac{m^2}{2}g_{\mu\nu}S = J_{\mu\nu}\,.$$

This can be converted to the following equivalent form

$$-D^\sigma D_\sigma S_{\mu\nu} - D_\mu D_\nu S_\sigma{}^\sigma + D_\mu D^\sigma S_{\sigma\nu} + D_\nu D^\sigma S_{\mu\sigma} - \\ - 2(S_{\mu\nu} - g_{\mu\nu}S_\sigma{}^\sigma) + m_1^2 S_{\mu\nu} + m_2^2 g_{\mu\nu}S^\sigma_\sigma = 2\tilde{J}_{\mu\nu}\,, \tag{4.5}$$

where $m_2^2 = m_1^2/(d-1) = m^2/(d-1)$ and $\tilde{J}_{\mu\nu} = J_{\mu\nu} - \frac{1}{d-1}J_\sigma{}^\sigma$. We work with arbitrary m_1 and m_2 and only use the relation $m_2^2 = m_1^2/(d-1)$ towards the end of the calculation.

We look for solutions of the form

$$S_{\mu\nu}(z) = \int d^{d+1}w\sqrt{g}G_{\mu\nu;\mu'\nu'}(z,w)J^{\mu'\nu'}(w)\,, \tag{4.6}$$

where $G_{\mu\nu;\mu'\nu'}(z,w)$is the bi-tensor propagator for the massive symmetric tensor. Inserting this expression in eq. (4.5), we obtain the equation

$$-D^\sigma D_\sigma G_{\mu\nu;\mu'\nu'} - D_\mu D_\nu G_\sigma{}^\sigma{}_{\mu'\nu'} + D_\mu D^\sigma G_{\sigma\nu;\mu'\nu'} + D_\nu D^\sigma G_{\mu\sigma;\mu'\nu'} - \\ - 2(G_{\mu\nu;\mu'\nu'} - g_{\mu\nu}G_\sigma{}^\sigma{}_{;\mu'\nu'}) + m_1^2 G_{\mu\nu;\mu'\nu'} + m_2^2 g_{\mu\nu}G_\sigma{}^\sigma{}_{;\mu'\nu'} = \\ = \Big(g_{\mu\mu'}g_{\nu\nu'} + g_{\mu\nu'}g_{\nu\mu'} - \frac{2}{d-1}g_{\mu\nu}g_{\mu'\nu'}\Big)\delta(z,w). \tag{4.7}$$

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The next step is to write $G_{\mu\nu;\mu'\nu'}(z,w)$ in terms of invariant bi-tensors in AdS_{d+1} and scalar functions. We will use the following five bi-tensors defined in [6]:

$$
\begin{aligned}
T^{(1)}_{\mu\nu;\mu'\nu'} &= g_{\mu\nu}\, g_{\mu'\nu'}\,, \\
T^{(2)}_{\mu\nu;\mu'\nu'} &= \partial_\mu u\, \partial_\nu u\, \partial_{\mu'} u\, \partial_{\nu'} u, \\
T^{(3)}_{\mu\nu;\mu'\nu'} &= \partial_\mu \partial_{\mu'} u\, \partial_\nu \partial_{\nu'} u + \partial_\mu \partial_{\nu'} u\, \partial_\nu \partial_{\mu'} u, \\
T^{(4)}_{\mu\nu;\mu'\nu'} &= g_{\mu\nu}\, \partial_{\mu'} u\, \partial_{\nu'} u + g_{\mu'\nu'}\, \partial_\mu u\, \partial_\nu u, \\
T^{(5)}_{\mu\nu;\mu'\nu'} &= \partial_\mu \partial_{\mu'} u\, \partial_\nu u \partial_{\nu'} u + \partial_\nu \partial_{\mu'} u\, \partial_\mu u \partial_{\nu'} u + (\mu' \leftrightarrow \nu')\,.
\end{aligned} \tag{4.8}
$$

Our ansatz for the propagator $G_{\mu\nu;\mu'\nu'}(z,w)$ is

$$
\begin{aligned}
G_{\mu\nu;\mu'\nu'} = {} & (\partial_\mu \partial_{\mu'} u\, \partial_\nu \partial_{\nu'} u + \partial_\mu \partial_{\nu'} u\, \partial_\nu \partial_{\mu'} u)\, G(u) + g_{\mu\nu} g_{\mu'\nu'}\, H(u) + \\
& + D_\mu L_{\nu;\mu'\nu'} + D_\nu L_{\mu;\mu'\nu'} + D_{\mu'} \Lambda_{\mu\nu;\nu'} + D_{\nu'} \Lambda_{\mu\nu;\mu'}\,,
\end{aligned}
$$

where

$$
\begin{aligned}
\Lambda_{\nu;\mu'\nu'} &= g_{\mu'\nu'} \partial_\nu u A(u) + \partial_{\mu'} u \partial_{\nu'} u \partial_\nu u C(u) + (\partial_{\mu'} \partial_\nu u \partial_{\nu'} u + \partial_{\nu'} \partial_\nu u \partial_{\mu'} u) B(u)\,, \\
\Lambda_{\mu\nu;\nu'} &= g_{\mu\nu} \partial_{\nu'} u A(u) + \partial_\mu u \partial_\nu u \partial_{\nu'} u C(u) + (\partial_\mu \partial_{\nu'} u \partial_\nu u + \partial_\nu \partial_{\nu'} u \partial_\mu u) B(u)\,,
\end{aligned} \tag{4.9}
$$

and

$$
\begin{aligned}
D_\mu L_{\nu;\mu'\nu'} + D_\nu L_{\mu;\mu'\nu'} = {} & 2(1+u) A T^{(1)} + g_{\mu'\nu'} \partial_\mu u \partial_\nu u 2A' + 2C' T^{(2)} + 2B T^{(3)} + C T^{(5)} + \\
& + g_{\mu\nu} \partial_{\mu'} u \partial_{\nu'} u (2(1+u)C + 4B) + B' T^{(5)}\,, \\
D_{\mu'} \Lambda_{\mu\nu;\nu'} + D_{\nu'} \Lambda_{\mu\nu;\mu'} = {} & 2(1+u) A T^{(1)} + g_{\mu\nu} \partial_{\mu'} u \partial_{\nu'} u 2A' + 2C' T^{(2)} + 2B T^{(3)} + C T^{(5)} + \\
& + g_{\mu'\nu'} \partial_\mu u \partial_\nu u (2(1+u)C + 4B) + B' T^{(5)}\,.
\end{aligned} \tag{4.10}
$$

This ansatz is invariant under exchange $z \leftrightarrow w$ (exchange of primed and unprimed indices). We note that

$$
\begin{aligned}
D_{\mu'} \Lambda_{\mu\nu;\nu'} + D_{\nu'} \Lambda_{\mu\nu;\mu'} = {} & D_\mu L_{\nu;\mu'\nu'} + D_\nu L_{\mu;\mu'\nu'} + \\
& + \Big(-2A' + 2(1+u)C + 4B\Big) g_{\mu'\nu'} \partial_\mu u \partial_\nu u - \\
& - \Big(-2A' + 2(1+u)C + 4B\Big) g_{\mu\nu} \partial_{\mu'} u \partial_{\nu'} u\,.
\end{aligned}
$$

The term $D_\mu L_{\nu;\mu'\nu'} + D_\nu L_{\mu;\mu'\nu'}$ just corresponds to diffeomorphisms in z and is annihilated by the "wave operator". Unlike the massless graviton case, all five functions $G(u)$, $H(u)$, $A(u)$, $B(u)$ and $C(u)$ are physical. We now find the differential equations

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for these five functions. Using this ansatz in (4.7) and using relations in eqs. (1.5), it is simple, but tedious to derive the following equation:

$$
\begin{aligned}
&-D^\sigma D_\sigma G_{\mu\nu;\mu'\nu'} - D_\mu D_\nu G_\sigma{}^\sigma{}_{\mu'\nu'} + D_\mu D^\sigma G_{\sigma\nu;\mu'\nu'} + D_\nu D^\sigma G_{\mu\sigma;\mu'\nu'} - \\
&\qquad -2(G_{\mu\nu;\mu'\nu'} - g_{\mu\nu} G_\sigma{}^\sigma{}_{;\mu'\nu'}) + m_1^2 G_{\mu\nu;\mu'\nu'} + m_2^2 g_{\mu\nu} G_\sigma{}^\sigma{}_{;\mu'\nu'} = \\
&= T^{(1)}\Big[-u(2+u)H'' - 2d(1+u)H' + 2dH + 4G - 2(1+u)G' + \\
&\qquad + 4(1+u)m_1^2 A + m_1^2 H + m_2^2(d+1)H + 4m_2^2(d+1)(1+u)A + \\
&\qquad + 2m_2^2 u(2+u)A' + 8m_2^2 B + 2m_2^2 G + \\
&\qquad + 2m_2^2 u(2+u)(1+u)C + 4m_2^2 u(2+u)B + \\
&\qquad + \Big((d+1)u^2 + (2d+2)u + d\Big)\Big(-4A' + 4(1+u)C + 8B\Big) + \\
&\qquad + u(2+u)(1+u)\Big(-2A'' + 2(1+u)C' + 2C + 4B'\Big) + \\
&\qquad + 4(1+u)C + 8B - 4A'\Big] + \\
&\quad + T^{(2)}\Big[-2G'' + 4m_1^2 C' - (d-1)(-2(1+u)C'' - 4C' - 4B'' + 2A''')\Big] + \\
&\quad + T^{(3)}\Big[-u(2+u)G'' - (d-1)(1+u)G' + 2(d-1)G + 4m_1^2 B + m_1^2 G - \\
&\qquad - (d-1)(-2(1+u)C - 4B + 2A')\Big] + \\
&\quad + g_{\mu\nu}\partial_{\mu'}u\partial_{\nu'}u\Big[2(1+u)G' + 4(d+1)G + 4m_1^2 B + 2(1+u)m_1^2 C + \\
&\qquad\qquad + 2m_2^2(1+u)(d+1)C + 2(m_1^2 + m_2^2(d+1))A' + \\
&\qquad\qquad + 8m_2^2(1+u)C + 8m_2^2(1+u)B' + 4m_2^2 u(2+u)C' + \\
&\qquad\qquad + 8m_2^2 B + 2m_2^2 G + 4m_2^2(d+1)B - \\
&\qquad\qquad - 2(d+2)(1+u)\Big(-2(1+u)C' - 2C - 4B' + 2A''\Big) - \\
&\qquad\qquad - u(2+u)\Big(-2(1+u)C'' - 4C' - 4B'' + 2A'''\Big) - \\
&\qquad\qquad - 2(d+1)\Big(-2(1+u)C - 4B + 2A'\Big)\Big] + \\
&\quad + g_{\mu'\nu'}\partial_\mu u\partial_\nu u\Big[-2G'' - (d-1)H'' + 2m_1^2 A' + 4m_1^2 B + 2m_1^2(1+u)C + \\
&\qquad\qquad + 2(d-1)\Big(-2A' + 2(1+u)C + 4B\Big) + \\
&\qquad\qquad + (1+u)(d-1)\Big(-2A'' + 2(1+u)C' + 2C + 4B'\Big)\Big] + \\
&\quad + T^{(5)}\Big[(1+u)G'' + (d-1)G' + 2m_1^2 B' + 2m_1^2 C + \\
&\qquad + (d-1)\Big(2(1+u)C' + 2C + 4B' - 2A''\Big)\Big].
\end{aligned}
\tag{4.11}
$$

We have dropped indices from $T^{(i)}$ for clarity. We define

$$
f \equiv -2A' + 2(1+u)C + 4B\,. \tag{4.12}
$$

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For z and w separate, we can set the scalar coefficient of each independent tensor to 0. We then obtain the following system of six differential equations:

$$\begin{aligned}-u(2+u)H'' - 2d(1+u)H' + 2dH + 4G - 2(1+u)G' +\\ +4(1+u)m_1^2 A + m_1^2 H + m_2^2(d+1)H + 4m_2^2(d+1)(1+u)A +\\ +2m_2^2 u(2+u)A' + 8m_2^2 B + 2m_2^2 G +\\ +2m_2^2 u(2+u)(1+u)C + 4m_2^2 u(2+u)B + 2((d+1)u^2 +\\ +(2d+2)u + d)f + u(2+u)(1+u)f' + 2f = 0\,,\end{aligned} \tag{4.13}$$

$$-2G'' + 4m_1^2 C' + (d-1)f'' = 0\,, \tag{4.14}$$

$$\begin{aligned}-u(2+u)G'' - (d-1)(1+u)G' +\\ +2(d-1)G + 4m_1^2 B + m_1^2 G + (d-1)f = 0\,,\end{aligned} \tag{4.15}$$

$$\begin{aligned}2(1+u)G' + 4(d+1)G + 4m_1^2 B + 2(1+u)m_1^2 C +\\ +2m_2^2(1+u)(d+1)C + 2(m_1^2 + m_2^2(d+1))A' + 8m_2^2(1+u)C +\\ +8m_2^2(1+u)B' + 4m_2^2 u(2+u)C' + 8m_2^2 B + 2m_2^2 G +\\ +4m_2^2(d+1)B + 2(d+2)(1+u)f' + u(2+u)f'' + 2(d+1)f = 0\,,\end{aligned} \tag{4.16}$$

$$\begin{aligned}-2G'' - (d-1)H'' + 2m_1^2 A' + 4m_1^2 B +\\ +2m_1^2(1+u)C + 2(d-1)f + (1+u)(d-1)f' = 0\,,\end{aligned} \tag{4.17}$$

$$(1+u)G'' + (d-1)G' + 2m_1^2 B' + 2m_1^2 C + (d-1)f' = 0\,. \tag{4.18}$$

This is an over-determined system since we have six equations for five functions which can be traced to the fact that eq. (4.5) does not possess the symmetry under exchange of z and w which the propagator itself possesses. So we will have to check that one of the above equations is redundant.

Eq. (4.14) can be integrated to give

$$(d-1)f' = 2G' - 4m_1^2 C\,, \tag{4.19}$$

In this and all that follows, we will consistently drop integration constants since we want our solution to approach 0 as $u \to \infty$. We can integrate eq. (4.19) to obtain

$$(d-1)f = 2G - 4m_1^2 D\,, \tag{4.20}$$

where

$$C = D'\,. \tag{4.21}$$

Eq. (4.20) can then be substituted in eq. (4.18) to obtain

$$B - D = -\frac{1}{2m_1^2}\Big((1+u)G' + dG\Big)\,. \tag{4.22}$$

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Eq. (4.15), can be brought into the following form using eq. (4.20) and eq. (4.22):

$$u(2+u)G''(u)+(d+1)(1+u)G'(u)-m_1^2G(u)=0\,. \tag{4.23}$$

This is an uncoupled differential equation for $G(u)$. In fact, this is just the equation for the scalar propagator with $m^2=m_1^2$. The solution is

$$\begin{aligned} G(u) &= \tilde{C}_\Delta(2u^{-1})^\Delta F\left(\Delta,\Delta-\frac{d}{2}+\frac{1}{2};2\Delta-d+1;-2u^{-1}\right),\\ \tilde{C}_\Delta &= \frac{\Gamma(\Delta)\Gamma(\Delta-d/2+1/2)}{(4\pi)^{(d+1)/2}\Gamma(2\Delta-d+1)}\,, \end{aligned} \tag{4.24}$$

where

$$\Delta=\frac{d}{2}+\frac{1}{2}\sqrt{d^2+4m_1^2}\,. \tag{4.25}$$

We now look at eq. (4.16) and use eq. (4.19), eq. (4.20) and eq. (4.23) to bring it to the following form:

$$\begin{aligned} &\Big(m_1^2-(d-1)m_2^2\Big)u(2+u)D''+\\ &+\Big(m_1^2-(d-1)m_2^2\Big)(d+5)(1+u)D'+\\ &+\Big(m_1^4-4m_1^2+m_1^2m_2^2(d+1)+2m_2^2(d^2+d-2)\Big)D=\\ &\qquad=-\frac{m_2^2}{m_1^2}(d-1)G''-\left(-2+\frac{m_2^2}{m_1^2}(d^2+d-2)\right)(1+u)G'-\\ &\qquad-\left(-2d+\frac{m_2^2}{m_1^2}d(d^2+d-2)+m_2^2d\right)G\,. \end{aligned}$$

We notice that for $m_1=(d-1)m_2^2$ eq. (4.26) becomes an algebraic equation.[4] In this case,[5]

$$D=\frac{1}{2d(d-1+m_1^2)m_1^2}\Big((d-1)G''+d(d-1)(1+u)G'+(d^3-d^2+m_1^2d)G\Big)\,. \tag{4.26}$$

From eq. (4.21), we can get an expression for $C(u)$:

$$C=\frac{1}{2d(d-1+m_1^2)m_1^2}\Big((d-1)G'''+d(d-1)(1+u)G''+(d^3-d+m_1^2d)G'\Big)\,, \tag{4.27}$$

and from eq. (4.22), we obtain

$$B=\frac{1}{2dm_1^2(d-1+m_1^2)}\Big((d-1)G''-m_1^2d(1+u)G'-m_1^2d(d-1)G\Big)\,. \tag{4.28}$$

[4]This relation between m_1 and m_2 was discussed after eq. (4.5).

[5]Actually, the expression for D in eq. (4.26) is a solution to eq. (4.26) for any m_1 and m_2. It is the unique solution for $m_1=(d-1)m_2^2$.

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It will be useful to obtain an expression for f from eq. (4.20):

$$f = -\frac{2}{d(d-1+m_1^2)}\Big(G'' + d(1+u)G' + d(d-1)G\Big)\,. \tag{4.29}$$

Using the definition of f (eq. (4.12)) and expressions for B and D, we obtain

$$\begin{aligned} A' = \frac{1}{2m_1^2 d(d-1+m_1^2)}\Big(&(d-1)(1+u)G''' + (d^2+d-2+2m_1^2)G'' - \\ &- m_1^2 d(1+u)G' + m_1^2 d(d-1)G\Big)\,. \end{aligned} \tag{4.30}$$

which can be integrated to give

$$\begin{aligned} A = \frac{1}{2m_1^2 d(d-1+m_1^2)}\Bigg(&(d-1)(1+u)G'' + (d^2-1+2m_1^2)G' + \\ &+ m_1^2 d(1+u)G + m_1^2 d(d-2)\int G\Bigg)\,. \end{aligned} \tag{4.31}$$

We will integrate this once more since the resulting expression for $\int A$ will be used in calculation of H.

$$\begin{aligned} \int A = \frac{1}{2m_1^2 d(d-1+m_1^2)}\Bigg(&(d-1)(1+u)G' + (d^2-d+2m_1^2)G' + \\ &+ m_1^2 d(1+u)\int G + m_1^2 d(d-3)\int\!\!\int G\Bigg)\,. \end{aligned} \tag{4.32}$$

Eq. (4.17) can be brought into the form:

$$(d-1)H = -2G + 2m_1^2\int A - 2m_1^2(1+u)\int D - 2(d-2)\int\!\!\int G\,, \tag{4.33}$$

from which it follows that

$$H = -\frac{2}{d(d-1+m_1^2)}\left((2d^2+m_1^2 d-4d)\int\!\!\int G + d(1+u)\int G + (d+m_1^2)G\right)\,. \tag{4.34}$$

We still need to check that eq. (4.13) is redundant. Using eqs. (4.19), (4.22), (4.34) and (4.21), we can rewrite eq. (4.13) in terms of $G(u)$ and $D(u)$ and their derivatives. Then, using eq. (4.23) and eq. (4.26) and integrated versions of these equations, it is tedious but straight forward to show that the left hand side is identically zero.

5. The limit as $u \to 0$

In this section, we will check the $u \to 0$ limit of the propagators for the massive vector and massive symmetric tensor in AdS_{d+1}. We have set the AdS scale R (the radius of curvature) to unity. Had not set R to unity and taken the limit $R \to \infty$

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in our propagators, we should naively recover the flat space results. This is almost true; since m in the AdS space is measured in units of $1/R$, to match to the flat space propagator of a non-zero mass, we also have to take $m \to \infty$. So if we take the $u \to 0$ and $m \to \infty$ the propagators should approach the short distance limit of the corresponding propagators in flat space.

For comparison with the flat space propagators, it is convenient to re-write the AdS_{d+1} propagators in terms of n_μ and $n_{\mu'}$ where

$$n_\mu(z,w) = D_\mu \mu(z,w)\,, \qquad n_{\mu'}(z,w) = D_{\mu'}\mu(z,w)\,,$$

$\mu(z,w)$ is the geodesic distance between z and w and $g_\mu{}^{\nu'}$ is the parallel transporter from z to w. We will also need the following dictionary [6]

$$\begin{aligned} u &= \cosh(\mu) - 1\,, \\ n_\mu &= \frac{\partial_\mu u}{\sqrt{u(u+2)}}\,, \\ g_{\mu\nu'} &= \partial_\mu \partial_{\nu'} u + \frac{\partial_\mu u \partial_{\nu'} u}{u+2}\,. \end{aligned} \tag{5.1}$$

5.1 Massive vector

For the massive vector, the propagator we obtained in AdS_{d+1} is (specializing the case of p-forms in section 3 to $p=1$):

$$G_{\mu\mu'}(z,w) = (F+L)\partial_\mu \partial_{\mu'} u + L' \partial_\mu u \partial_{\mu'} u\,. \tag{5.2}$$

In terms of n_μ and $n_{\mu'}$, we can re-write $G_{\mu\mu'}(z,w)$ as

$$G_{\mu\mu'}(z,w) = (F+L) g_{\mu\mu'} - \Big(u(2+u)L' - u(F+L)\Big) n_\mu n_{\mu'}\,.$$

As $u \to 0$,

$$G_{\mu\mu'}(z,w) = -\frac{1}{m^2}\mathcal{F}' g_{\mu\mu'} + \frac{2}{m^2} u \mathcal{F}'' n_\mu n_{\mu'}\,. \tag{5.3}$$

where $\mathcal{F}$ is just the leading term of F as $u \to 0$.[6]

In flat space, the propagator for the massive vector is given by[7]

$$\begin{aligned} G^{flat}_{\mu\mu'} &= (g_{\mu\mu'} + \partial_\mu \partial_{\mu'}) N(r^2) \\ &= \left(1 - \frac{2}{m^2} N'\right) g_{\mu\mu'} + \frac{4}{m^2} r^2 N'' n_\mu n_{\nu'}\,, \end{aligned}$$

where $r^2 = (x-y)^2$ and

$$N(r^2) = \int \frac{d^{d+1}k}{(2\pi)^{\frac{d+1}{2}}} \frac{e^{ik.(x-y)}}{k^2 + m^2}\,.$$

[6] $\mathcal{F} \sim 1/u^{\frac{d-1}{2}}$.

[7] $N'(r^2)$ denotes $\frac{d}{d(r^2)} N(r^2)$

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As $u \to 0$, this approaches

$$G^{flat}_{\mu\mu'} = -\frac{2}{m^2}\mathcal{N}'(r^2) g_{\mu\mu'} + \frac{4}{m^2} r^2 \mathcal{N}''(r^2) n_\mu n_{\nu'} \,. \tag{5.4}$$

$\mathcal{N}$ is the leading term in $N(r^2)$ as $r^2 \to 0$.[8] Also for small u, $u \approx \mu^2/2 = r^2/2$ (from eq. (5.1)).

The normalization of $F(u)$ in eq. (5.2) was chosen such that it matches with $N(r^2)$ as $u \to 0$, i.e.

$$\mathcal{F}(u) = \mathcal{N}(r^2) = \mathcal{N}(2u) \,.$$

In terms of u, as $u \to 0$,

$$G^{flat}_{\mu\mu'} = -\frac{1}{m^2}\frac{d}{du}\mathcal{N}(2u) g_{\mu\mu'} + \frac{2}{m^2} u \frac{d^2}{du^2}\mathcal{N}(2u) n_\mu n_{\nu'} \,, \tag{5.5}$$

which is exactly the same as the short distance limit in eq. (5.3).

5.2 Massive symmetric tensor

As $u \to 0$, $G_{\mu\nu;\mu'\nu'}(z,w)$ for the massive symmetric tensor should approach the short-distance limit of the massive symmetric tensor propagator in flat space. To check this, it is convenient re-write our expression for $G_{\mu\nu;\mu'\nu'}(z,w)$ in terms of tensors $\mathcal{O}^i$'s defined in [6]:

$$\begin{aligned}
\mathcal{O}^{(1)}_{\mu\nu\mu'\nu'} &= g_{\mu\nu} g_{\mu'\nu'} \,, \\
\mathcal{O}^{(2)}_{\mu\nu\mu'\nu'} &= n_\mu n_\nu n_{\mu'} n_{\nu'} \,, \\
\mathcal{O}^{(3)}_{\mu\nu\mu'\nu'} &= g_{\mu\mu'} g_{\nu\nu'} + g_{\mu\nu'} g_{\nu\mu'} \,, \\
\mathcal{O}^{(4)}_{\mu\nu\mu'\nu'} &= g_{\mu\nu} n_{\mu'} n_{\nu'} + g_{\mu'\nu'} n_\mu n_\nu \,, \\
\mathcal{O}^{(5)}_{\mu\nu\mu'\nu'} &= g_{\mu\mu'} n_\nu n_{\nu'} + g_{\mu\nu'} n_\nu n_{\mu'} + g_{\nu\nu'} n_\mu n_{\mu'} + g_{\nu\mu'} n_\mu n_{\nu'} \,.
\end{aligned} \tag{5.6}$$

We can write the tensors T_i defined earlier in terms of $\mathcal{O}_i$ by using eq. (5.1):

$$\begin{aligned}
T^{(1)} &= \mathcal{O}^{(1)} \,, \\
T^{(2)} &= [u(u+2)]^2 \mathcal{O}^{(2)} \,, \\
T^{(3)} &= 2u^2 \mathcal{O}^{(2)} + \mathcal{O}^{(3)} - u\mathcal{O}^{(5)} \,, \\
T^{(4)} &= u(u+2)\mathcal{O}^{(4)} \,, \\
T^{(5)} &= 4u^2(u+2)\mathcal{O}^{(2)} - u(u+2)\mathcal{O}^{(5)} \,.
\end{aligned}$$

[8] $\mathcal{N} \sim 1/r^{d-1}$.

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$G_{\mu\nu;\mu'\nu'}(z,w)$ can be written in terms of $\mathcal{O}^{(i)}$:

$$\begin{aligned}G_{\mu\nu;\mu'\nu'}(z,w) = &\Big(H+4(1+u)A\Big)\mathcal{O}^{(1)}+\Big(G+4B\Big)\mathcal{O}^{(3)}+\\ &+\Big(4u^2(u+2)^2C'+2u^2G+8u^2B+8u^2(u+2)C+8u^2(u+2)B'\Big)\mathcal{O}^{(2)}+\\ &+\Big(2A'+2(1+u)C+4B\Big)u(2+u)\mathcal{O}^{(4)}-\\ &-\Big((G+4B)u+(2C+2B')u(2+u)\Big)\mathcal{O}^{(5)}.\end{aligned}$$

We now take the $u\to 0$ and $m\to\infty$ limit:

$$\begin{aligned}G_{\mu\nu;\mu'\nu'}(z,w) = \frac{1}{2dm^4}\Big[&4(d-1)\mathcal{G}''\mathcal{O}_1+4(d-1)\mathcal{G}''\mathcal{O}_3+16(d-1)u^2\mathcal{G}''''\mathcal{O}_2+\\ &+8(d-1)u\mathcal{G}'''\mathcal{O}_4-8(d-1)u\mathcal{G}'''\mathcal{O}_5\Big].\end{aligned}$$

Here, $\mathcal{G}$ is the leading term in $G(u)$ as $u\to 0$.[9]

In flat euclidean space, the propagator for the massive symmetric tensor is calculated in the appendix. The short distance limit of the flat space propagator is given by the expression:

$$\begin{aligned}G^{flat}_{\mu\nu;\mu'\nu'}(x,y) = &\frac{8(d-1)}{m^4d}\mathcal{N}''\mathcal{O}^{(1)}+\frac{32(d-1)}{m^4d}r^4\mathcal{N}''''\mathcal{O}^{(2)}+\frac{8(d-1)}{m^4d}\mathcal{N}''\mathcal{O}^{(3)}+\\ &+\frac{16(d-1)}{m^4d}r^2\mathcal{N}'''\mathcal{O}^{(4)}-\frac{16(d-1)}{m^4d}r^2\mathcal{N}'''\mathcal{O}^{(5)}.\end{aligned}$$

where $\mathcal{N}$ is, as before, the leading term in $N(r^2)$. $G(u)$ was normalized in eq. (4.24) such that

$$\mathcal{G}(u)=\mathcal{N}(r^2)=\mathcal{N}(2u)\,,$$

as $u=r^2/2\to 0$. Then, in terms of u, we can write eq. (5.7) as

$$\begin{aligned}G^{flat}_{\mu\nu;\mu'\nu'}(z,w) = \frac{1}{2dm^4}\Bigg[&4(d-1)\frac{d^2}{du^2}\mathcal{N}(2u)\mathcal{O}^{(1)}+4(d-1)\frac{d^2}{du^2}\mathcal{N}(2u)\mathcal{O}^{(3)}+\\ &+16(d-1)u^2\frac{d^4}{du^4}\mathcal{N}(2u)\mathcal{O}^{(2)}+8(d-1)u\frac{d^3}{du^3}\mathcal{N}(2u)\mathcal{O}^{(4)}-\\ &-8(d-1)u\frac{d^3}{du^3}\mathcal{N}(2u)\mathcal{O}^{(5)}\Bigg].\end{aligned}$$

This exactly matches the short distance limit of the propagator in AdS_{d+1} in eq. (5.7).

6. Summary of results

We have calculated the propagators for the massive symmetric tensor and p-form fields in AdS_{d+1} using the method developed in [6]. In this section, we summarize our results.

[9] $\mathcal{G}\sim 1/u^{\frac{d-1}{2}}$.

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p-forms. We defined two independent bi-tensors:

$$T_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} = \partial_{[\mu_1}\partial^{\mu_1'}u\partial_{\mu_2}\partial^{\mu_2'}u\cdots\partial_{\mu_p]}\partial^{\mu_2'}u\,,$$
$$S_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} = \partial_{[\mu_1}u\partial^{[\mu_1'}u\partial_{\mu_2}\partial^{\mu_2'}u\cdots\partial_{\mu_p]}\partial^{\mu_2']}u\,.$$

Then,

$$G_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} = (F(u)+pL(u))T_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'} + L'(u)S_{\mu_1\mu_2\cdots\mu_p}{}^{\mu_1'\mu_2'\cdots\mu_p'}\,.$$
$$F(u) = \tilde{C}_\Delta(2u^{-1})^\Delta F\left(\Delta,\Delta-\frac{d}{2}+\frac{1}{2};2\Delta-d+1;-2u^{-1}\right),$$
$$\tilde{C}_\Delta = \frac{\Gamma(\Delta)\Gamma(\Delta-d/2+1/2)}{(4\pi)^{(d+1)/2}\Gamma(2\Delta-d+1)}\,,$$

where

$$\Delta = \frac{d}{2}+\frac{1}{2}\sqrt{d^2+4m^2-4p(d-p)}\,,$$
$$L(u) = -\frac{1}{m^2}\Big((d-p)F(u)+(1+u)F'(u)\Big)\,.$$

Massive symmetric tensor. In terms of $T^{(i)}$ defined in eq. (4.8),

$$\begin{aligned}G_{\mu\nu;\mu'\nu'}(z,w) = {}&\Big(H(u)+4(1+u)A(u)\Big)T^{(1)}_{\mu\nu;\mu'\nu'} + 4C'(u)T^{(2)}_{\mu\nu;\mu'\nu'} + \\ &+\Big(G(u)+4B(u)\Big)T^{(3)}_{\mu\nu;\mu'\nu'} + \\ &+\Big(2A'(u)+2(1+u)C(u)+4B(u)\Big)T^{(4)}_{\mu\nu;\mu'\nu'} + \\ &+\Big(2C(u)+2B'(u)\Big)T^{(5)}_{\mu\nu;\mu'\nu'}\,.\end{aligned}$$

This can also be written in terms of $\mathcal{O}^{(i)}$ (eq. (5.6),

$$\begin{aligned}G_{\mu\nu;\mu'\nu'}(z,w) = {}&\Big(H+4(1+u)A\Big)\mathcal{O}^{(1)} + \Big(G+4B\Big)\mathcal{O}^{(3)} + \\ &+\Big(4u^2(u+2)^2C'+2u^2G+8u^2B+8u^2(u+2)C+8u^2(u+2)B'\Big)\mathcal{O}^{(2)} + \\ &+\Big(2A'+2(1+u)C+4B\Big)u(2+u)\mathcal{O}^{(4)} - \\ &-\Big((G+4B)u+(2C+2B')u(2+u)\Big)\mathcal{O}^{(5)}\,.\end{aligned}$$

We found,

$$G(u) = \tilde{C}_\Delta(2u^{-1})^\Delta F\left(\Delta,\Delta-\frac{d}{2}+\frac{1}{2};2\Delta-d+1;-2u^{-1}\right),$$

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where

$$\begin{aligned}
\Delta &= \frac{d}{2} + \frac{1}{2}\sqrt{d^2+4m^2}\,. \\
C(u) &= \frac{1}{2d(d-1+m_1^2)m_1^2}\Big((d-1)G''' + d(d-1)(1+u)G'' + (d^3-d+m_1^2 d)G'\Big)\,, \\
B(u) &= \frac{1}{2dm_1^2(d-1+m_1^2)}\Big((d-1)G'' - m_1^2 d(1+u)G' - m_1^2 d(d-1)G\Big)\,, \\
A(u) &= \frac{1}{2m_1^2 d(d-1+m_1^2)}\Big((d-1)(1+u)G'' + (d^2-1+2m_1^2)G' + \\
&\qquad\qquad + m_1^2 d(1+u)G + m_1^2 d(d-2)\int G\Big)\,, \\
H(u) &= -\frac{2}{d(d-1+m_1^2)}\Big((2d^2+m_1^2 d-4d)\iint G + d(1+u)\int G + (d+m_1^2)G\Big)\,.
\end{aligned}$$

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A. Massive symmetric tensor propagator in R^{d+1}

In this section, we will calculate the graviton propagator in flat space. The equation of motion is

$$-\partial^\sigma\partial_\sigma S_{\mu\nu} - \partial_\mu\partial_\nu S_\sigma{}^\sigma + \partial_\mu\partial^\sigma S_{\sigma\nu} + \partial_\nu\partial^\sigma S_{\sigma\mu} + m_1^2 S_{\mu\nu} + m_2^2 g_{\mu\nu} S_\sigma{}^\sigma = \tilde{T}_{\mu\nu}\,. \qquad \text{(A.1)}$$

where $\tilde{T}_{\mu\nu} = T_{\mu\nu} - \frac{1}{d-1} g_{\mu\nu} T_\sigma{}^\sigma$. The Green's function is defined by the equation

$$S_{\mu\nu} = \int d^{d+1}y G_{\mu\nu;\mu'\nu'}(x,y) T^{\mu'\nu'}(y)\,.$$

We will work in momentum space. Let

$$S_{\mu\nu} = \int \frac{d^{d+1}k}{(2\pi)^{\frac{d+1}{2}}} s_{\mu\nu}(k) e^{ikx}\,,$$

and

$$T_{\mu\nu} = \int \frac{d^{d+1}k}{(2\pi)^{\frac{d+1}{2}}} t_{\mu\nu}(k) e^{ikx}\,.$$

Then

$$s_{\mu\nu} = G_{\mu\nu;\mu'\nu'}(k) t^{\mu'\nu'}\,.$$

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In momentum space eq. (A.1) becomes:

$$k^2 G_{\mu\nu;\mu'\nu'} + k_\mu k_\nu G_\sigma{}^\sigma{}_{;\mu'\nu'} - k_\mu k^\sigma G_{\sigma\nu;\mu'\nu'} - \tag{A.2}$$
$$- k_\nu k^\sigma G_{\sigma\mu;\mu'\nu'} + m_1^2 k_\mu k^\sigma G_{\sigma\nu;\mu'\nu'} + m_2^2 g_{\mu\nu} G_\sigma{}^\sigma{}_{;\mu'\nu'} = g_{\mu\mu'} g_{\nu\nu'} + g_{\nu\mu'} g_{\mu\nu'} - \frac{2}{d-1} g_{\mu\nu} g_{\mu'\nu'} \,.$$

We will use the following ansatz for $G_{\mu\nu;\mu'\nu'}$:

$$\begin{aligned} G_{\mu\nu;\mu'\nu'} \;=\;& A(k^2) g_{\mu\nu} g_{\mu'\nu'} + B(k^2)(g_{\mu\mu'} g_{\nu\nu'} + g_{\nu\mu'} g_{\mu\nu'}) + C(k^2)(g_{\mu\nu} k_{\mu'} k_{\nu'} + g_{\mu'\nu'} k_\mu k_\nu) + \\ & + F(k^2)(g_{\mu\nu'} k_{\mu'} k_\nu + g_{\nu\nu'} k_{\mu'} k_\mu + g_{\mu\mu'} k_\nu k_{\nu'} + g_{\nu\mu'} k_\mu k_{\nu'}) + G(k^2) k_\mu k_{\mu'} k_\nu k_{\nu'} \,. \end{aligned}$$

Using this ansatz in eq. (A.3), we get

$$\begin{aligned} &\Big((k^2 + m^2 + m_2^2(d+1))A(k^2) + 2m_2^2 B(k^2) + m_2^2 k^2 C(k^2)\Big) g_{\mu\nu} g_{\mu'\nu'} + \\ &\quad + (k^2 + m^2) B(k^2)(g_{\mu\mu'} g_{\nu\nu'} + g_{\nu\mu'} g_{\mu\nu'}) + \\ &+ \Big((k^2 + m_1^2 + (d+1)m_2^2) C(k^2) + 4m_2^2 F(k^2) + m_2^2 k^2 G(k^2)\Big) g_{\mu\nu} k_{\mu'} k_{\nu'} + \\ &\quad + \Big(m_1^2 C(k^2) + (d-1)A(k^2) + 2B(k^2)\Big) g_{\mu'\nu'} k_\mu k_\nu + \\ &+ \Big(m_1^2 F(k^2) - B(k^2)\Big)(g_{\mu\nu'} k_{\mu'} k_\nu + g_{\nu\nu'} k_{\mu'} k_\mu + g_{\mu\mu'} k_\nu k_{\nu'} + g_{\nu\mu'} k_\mu k_{\nu'}) + \\ &\quad + \Big(m_1^2 G(k^2) + (d-1) C(k^2)\Big) k_\mu k_{\mu'} k_\nu k_{\nu'} = \\ &\quad = (g_{\mu\mu'} g_{\nu\nu'} + g_{\nu\mu'} g_{\mu\nu'}) - \frac{2}{d-1} g_{\mu\nu} g_{\mu'\nu'} \,. \end{aligned}$$

This gives a system of linear equations for the functions $A(k^2)$, $B(k^2)$, $C(k^2)$, $F(k^2)$, $G(k^2)$ which can be easily solved:

$$A(k^2) = -\frac{2}{d(k^2+m^2)} \,, \qquad B(k^2) = \frac{1}{k^2+m^2} \,, \qquad C(k^2) = -\frac{2}{m^2 d(k^2+m^2)} \,,$$
$$F(k^2) = \frac{1}{m^2(k^2+m^2)} \,, \qquad G(k^2) = \frac{2(d-1)}{m^4 d(k^2+m^2)} \,.$$

In position space, defining $r^2 = (x-y)_\mu (x-y)^\mu$,

$$\begin{aligned} G_{\mu\nu;\mu'\nu'}(x,y) = \bigg(& -\frac{2}{d} g_{\mu\nu} g_{\mu'\nu'} + g_{\mu\mu'} g_{\nu\nu'} + g_{\nu\mu'} g_{\mu\nu'} + \frac{2}{m^2 d}(g_{\mu\nu} \partial_{\mu'} \partial_{\nu'} + g_{\mu'\nu'} \partial_\mu \partial_\nu) + \\ & + \frac{1}{m^2}(g_{\mu\nu'} \partial_{\mu'} \partial_\nu + g_{\nu\nu'} \partial_{\mu'} \partial_\mu + g_{\mu\mu'} \partial_\nu \partial_{\nu'} + g_{\nu\mu'} \partial_\mu \partial_{\nu'}) + \\ & + \frac{2(d-1)}{m^4 d} \partial_\mu \partial_{\mu'} \partial_\nu \partial_{\nu'} \bigg) N(r^2) \end{aligned}$$

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$$
\begin{aligned}
&= \left(-\frac{2}{d}+\frac{8(d-1)}{m^4 d}N''+\frac{8}{m^2 d}N'\right)\mathcal{O}^{(1)}+\frac{32(d-1)}{m^4 d}r^4 N''''\mathcal{O}^{(2)}+\\
&\quad+\left(1-\frac{4}{m^2}N'+\frac{8(d-1)}{m^4 d}N''\right)\mathcal{O}^{(3)}+\\
&\quad+\left(\frac{8}{m^2 d}r^2 N''+\frac{16(d-1)}{m^4 d}r^2 N'''\right)\mathcal{O}^{(4)}+\\
&\quad+\left(\frac{4}{m^2}r^2 N''-\frac{16(d-1)}{m^4 d}r^2 N'''\right)\mathcal{O}^{(5)}\,.
\end{aligned}
$$

Here, $\mathcal{O}^{(i)}$'s are defined in eqs. (5.6). In the flat space case, $n_\mu = (x-y)_\mu/r$ and $n_{\mu'} = -(x-y)_{\mu'}/r$.

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